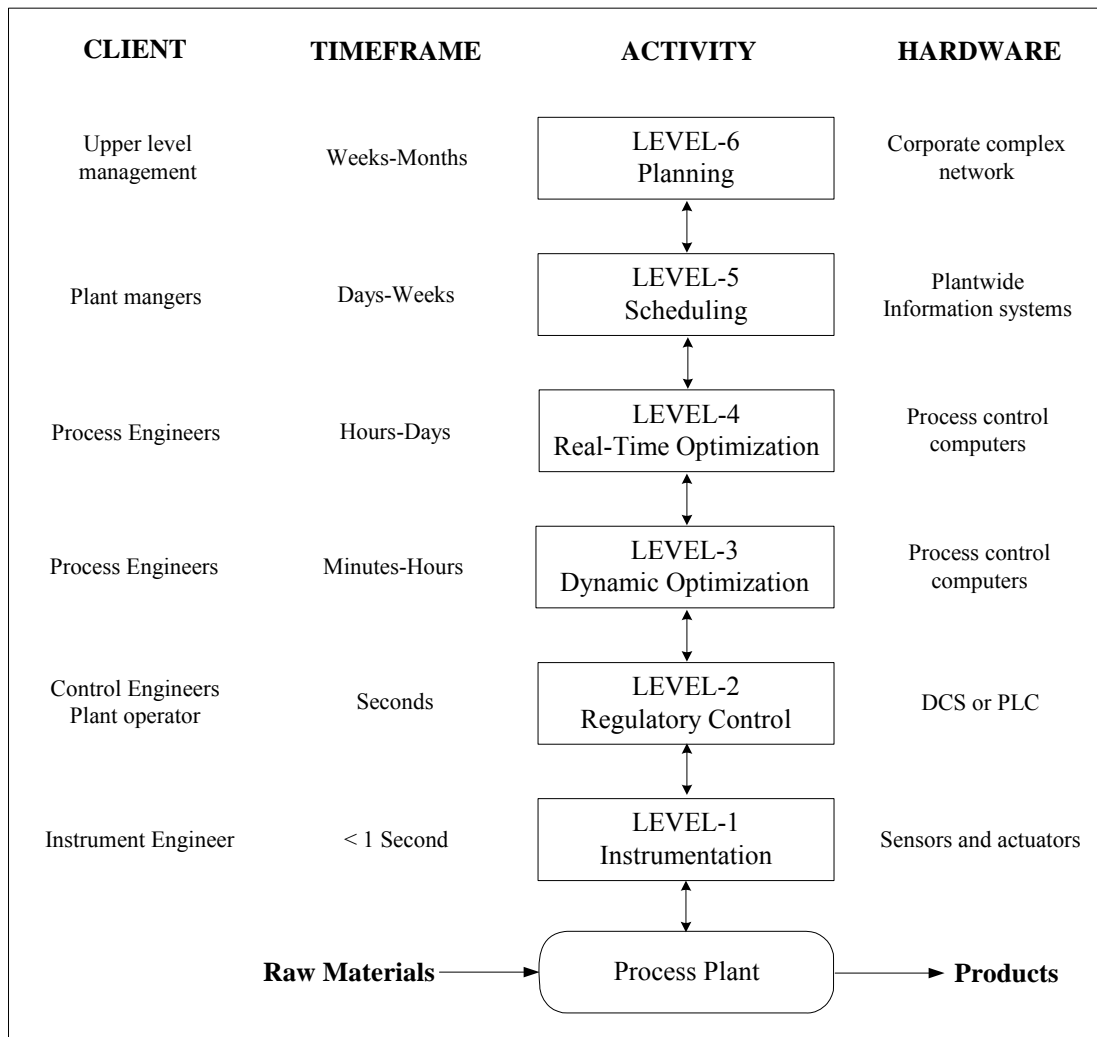
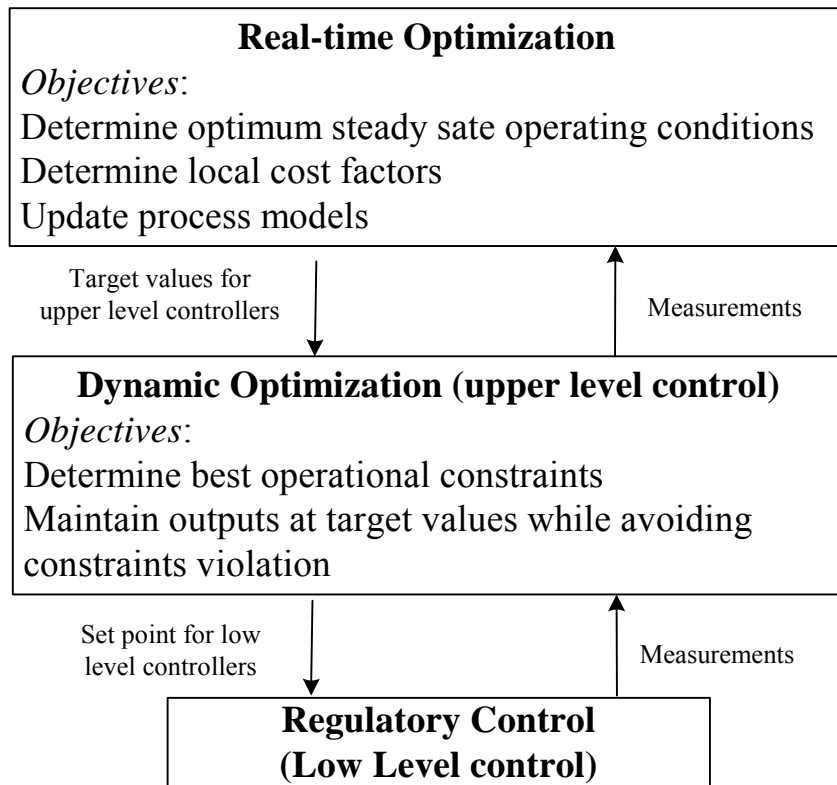
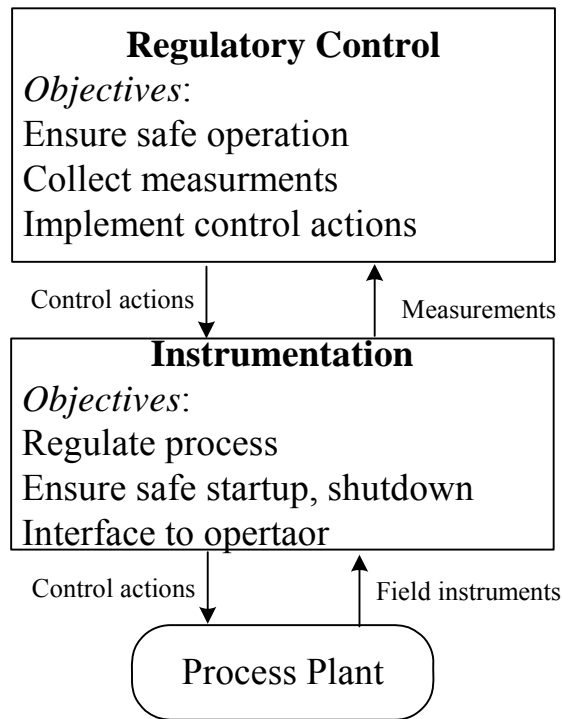


# CHAPTER 1:

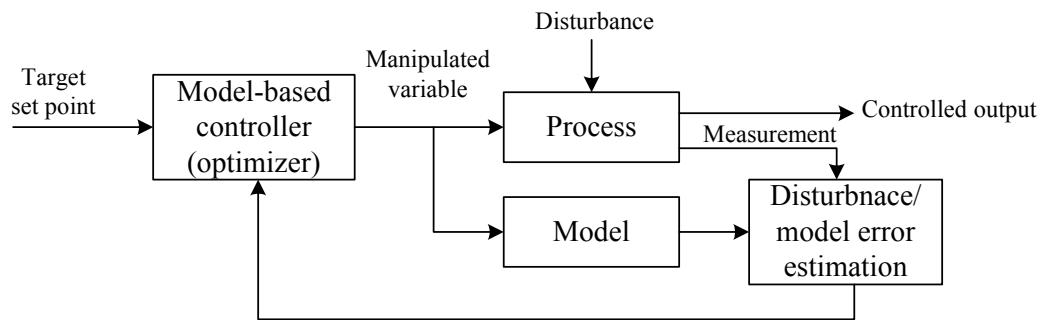
## PEREVIEW

### 1. Nature of Process Control Problem





## 2. Model-Based Controller (optimizer)



The major blocks associated with Model-based controller:

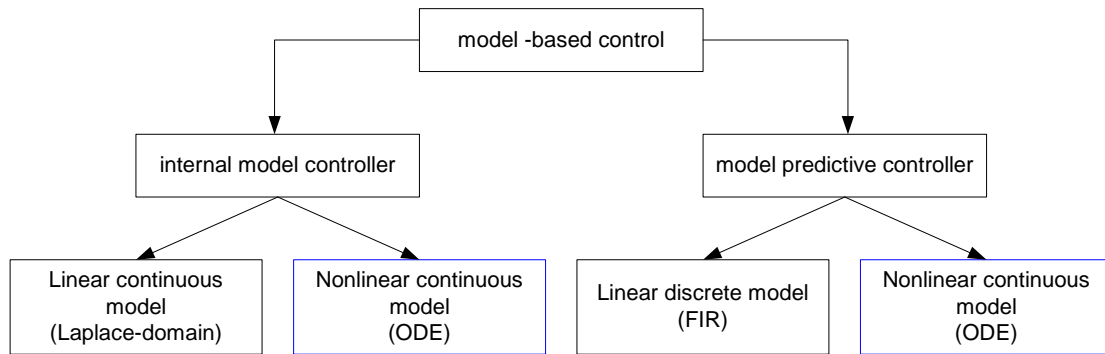
1. The process model
2. Disturbance /model error estimation
3. Controller or optimizer

*The major drawback of model-based controller is the accuracy of the model.*

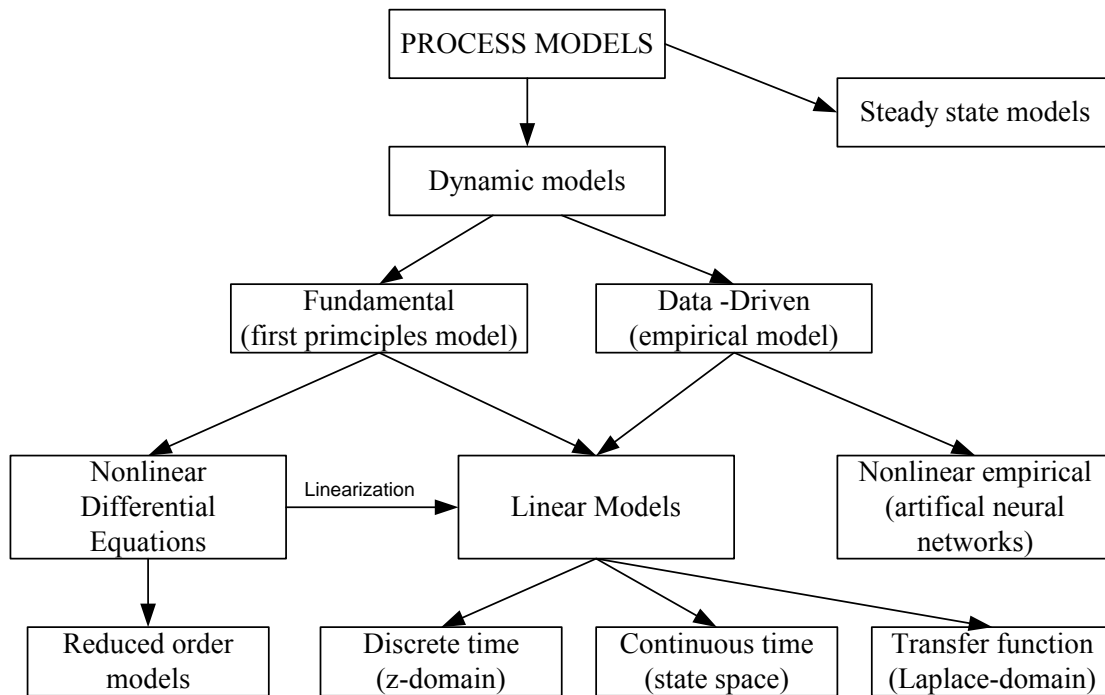
The process model is used at almost all layers of the hierarchal structure:

- Regulatory control: Models are used to tune PID controller, feed-forward, inferential controller
- Dynamic optimization: Models are used to predict the behavior of the process in the future, compute control actions.
- Real-time (steady state) optimization: Models are used to compute steady state operating conditions.
- Linear models are also used for planning and scheduling.

### 3. Types of Model-Based Controller



### 4. Types of Models



#### 4.1. Time Domain models (state Space)

Nonlinear state space:

$$\frac{dx}{dt} = f(x, u, w)$$
$$y = g(x, u)$$

Linear state space:

$$\frac{dx}{dt} = Ax(t) + Bu(t) + Dw(t)$$
$$y = Cx(t)$$

#### 4.2 Laplace Domain models

$$sX(s) = AX(s) + BU(s) + DW(s)$$
$$Y(s) = CX(s)$$

$$Y(s) = C(sI - A)^{-1}BU(s) + C(sI - A)^{-1}DW(s) \equiv G_p(s)u(s) + G_w(s)w(s)$$

$$G(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

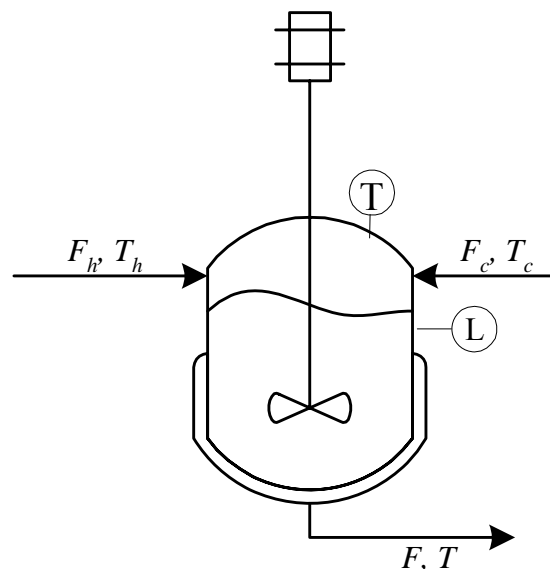
#### 4.3 Building State Space model from first principles

*Example:* Mixing Tank

**Mass Balance:**

$$A\rho \frac{dh}{dt} = F_c \rho_c + F_h \rho_h - F\rho$$

$$A \frac{dh}{dt} = F_c + F_h - F$$



### Energy Balance:

$$\frac{d}{dt} \rho A h C_p T = \rho F_c C_p T_c + \rho F_h C_p T_h - \rho F C_p T$$

$$A \frac{d}{dt} h T = F_c T_c + F_h T_h - F T$$

$$A h \frac{dT}{dt} + A T \frac{dh}{dt} = F_c T_c + F_h T_h - F T$$

$$A h \frac{dT}{dt} = F_c (T_c - T) + F_h (T_h - T)$$

The state space equations:

$$A \frac{dh}{dt} = F_c + F_h - F \equiv f_1(F_c, F_h, F)$$

$$A h \frac{dT}{dt} = F_c (T_c - T) + F_h (T_h - T) \equiv f_2(F_c, F_h, T_c, T_h, T)$$

Define:

$$\mathbf{x} = \begin{bmatrix} h \\ T \end{bmatrix}; \quad \mathbf{u} = \begin{bmatrix} F_c \\ F_h \end{bmatrix}; \quad \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Linearize:

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} 0 & 0 \\ 0 & -(F_c + F_h) \end{bmatrix}_{ss}$$

$$\mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}} = \begin{bmatrix} 1 & 1 \\ T_c - T & T_h - T \end{bmatrix}_{ss}$$

$$\mathbf{D} = \frac{\partial \mathbf{f}}{\partial w} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{ss}$$

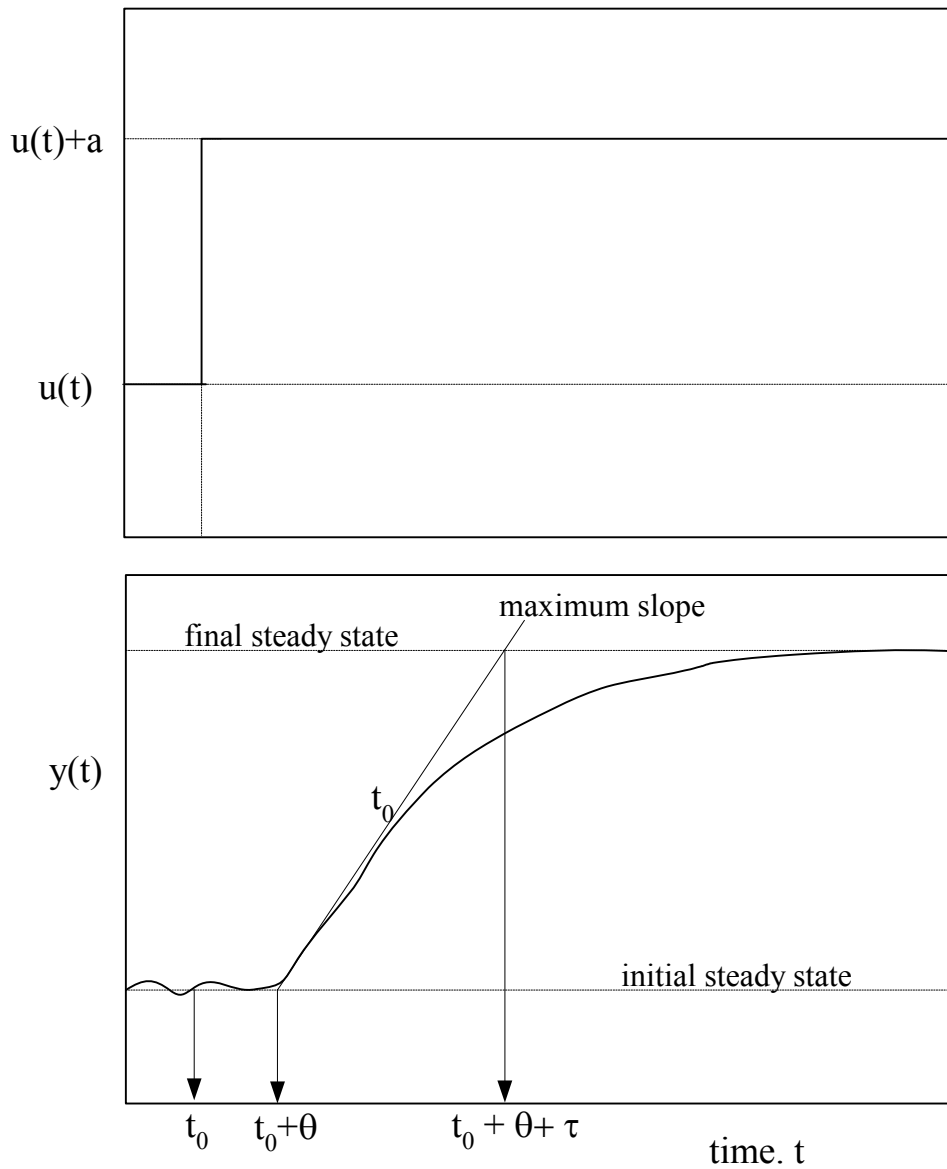
$$C = [1 \quad 1]$$

$$\frac{dx}{dt} = \mathbf{Ax}(t) + \mathbf{Bu}(t)$$

$$\mathbf{y} = \mathbf{Cx}(t)$$

### 3.4 Identifying a first-order process with dead time

$$G(s) = \frac{ke^{-\theta s}}{\tau s + 1}$$



## 5. Properties of Transfer Function

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m}{a_n} \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

*Pole*: is the root of the denominator of the transfer function, i.e. the root of the characteristic polynomial. It directly determines:

- The stability of the system (positive poles)
- The potential of periodic transient (imaginary poles)

*Zero*: is the root of the numerator of the transfer function. It determines an inverse response (positive zero).

*Casuality*: A physical system is *causal* when the order of the denominator is greater than the numerator, and when the transfer function goes to 0 as  $s \rightarrow \infty$ , the system is hence *strictly proper*. If the transfer function contains  $e^{\theta s}$  or the order of numerator is higher than the denominator, then the system is *non-causal* or *not realizable* because the current values of the system depends on the future values of the variables.

*Steady state gain*: is the steady state value of the transfer function, is evaluated by setting  $s = 0$  in the stable transfer function.

### 5.1 Effect of poles and zeros

The poles and zeros of a transfer function affect the dynamic of a process.

Consider a particular transfer function:

$$G(s) = \frac{K}{s(\tau_1 s + 1)(\tau^2 s^2 + 2\zeta\tau s + 1)}$$

The poles, i.e. the roots of the **characteristic equation** are:

$$s_1 = 0$$

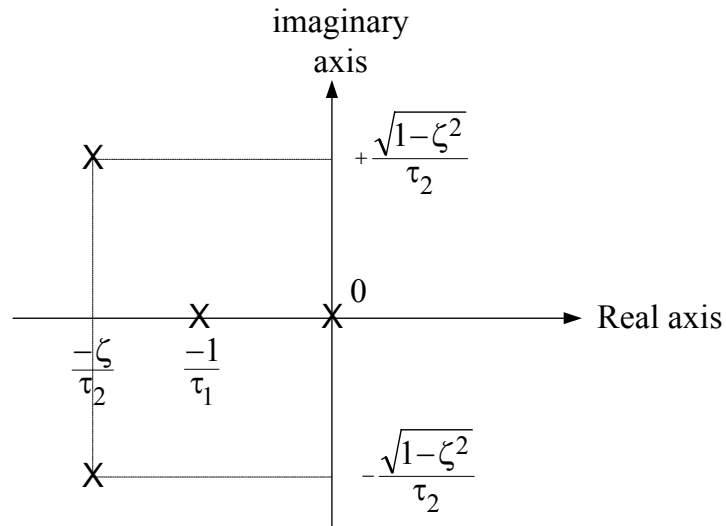
$$s_2 = -\frac{1}{\tau_1}$$

$$s_3 = -\frac{\zeta}{\tau_2} + j \frac{\sqrt{1 - \zeta^2}}{\tau_2}$$

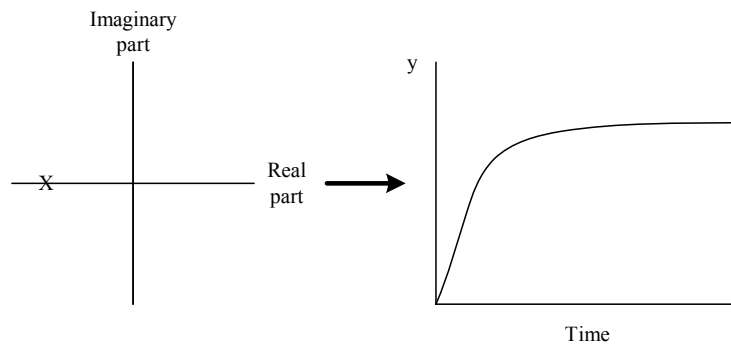
$$s_4 = -\frac{\zeta}{\tau_2} - j \frac{\sqrt{1 - \zeta^2}}{\tau_2}$$



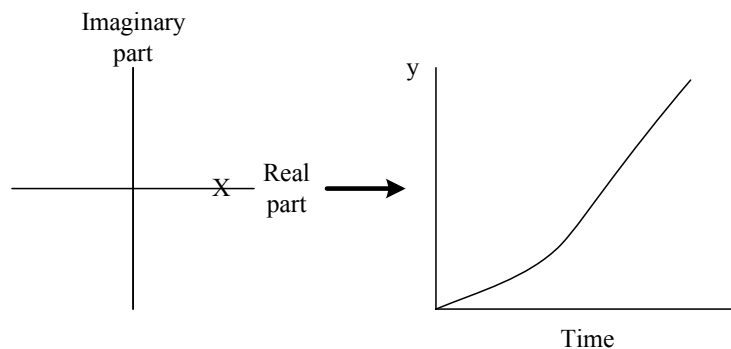
The poles can be represented in the complex plane as follows:



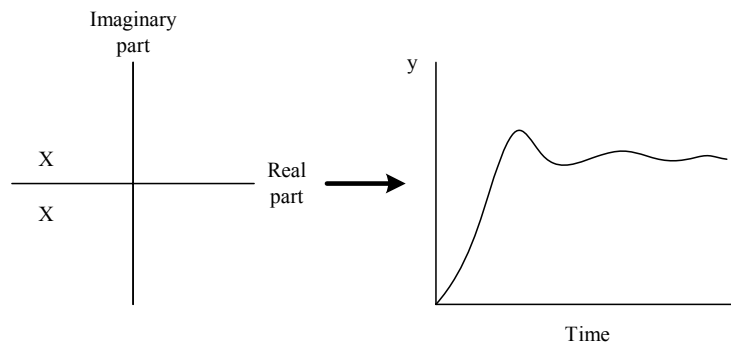
- Complex poles indicate the response will contain sine and cosine modes, i.e. will exhibit oscillation.
- Negative poles will result in a stable decaying response.
- Positive poles indicate that the response will have unstable mode.



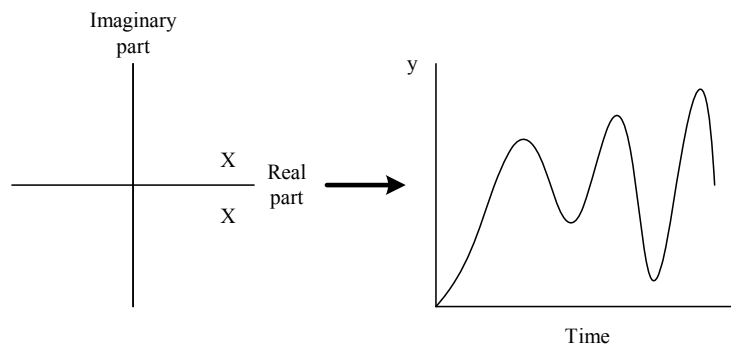
Negative real root



Positive real root



Complex roots with negative real part



Complex roots with positive real part

- A process with RHP zeros is called non-minimum-phase
- A process with odd number of RHP zeros has an inverse response.