CHAPTER 4:

2DF DESIGN OF IMC CONTROLLER

4.1 The Structure of 2DF IMC Design

- In the 1DF design, the disturbance is assumed to be added directly to the output without dynamics.
- In the 2DF design, the disturbance is assumed to enter through the process dynamics.

Figure 1 below shows the structure of the 2DF IMC controller.

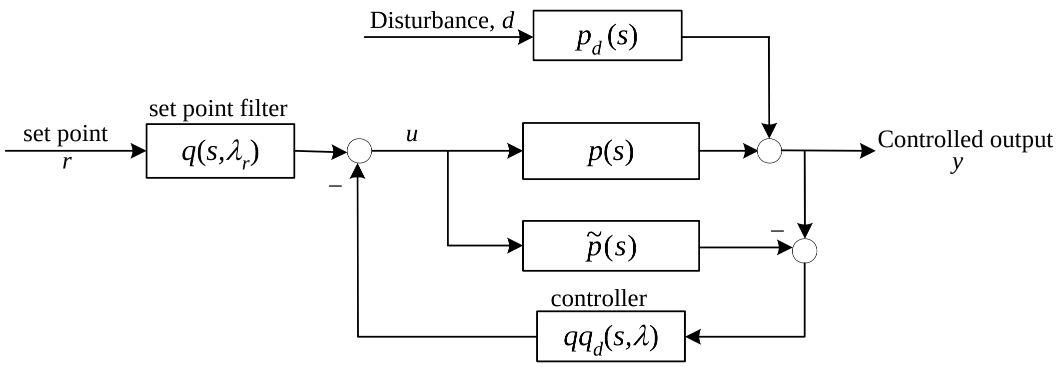


Figure 1: 2DF IMC block diagram

The transfer functions are as follows (see proof at Appendix):

Closed-loop response:

$$y(s)=\frac{p(s)q(s)}{1+(p(s)-\tilde{p}(s))qq_d(s)}r(s)+\frac{(1-p(s)qq_d(s))p_d(s)}{1+(p(s)-\tilde{p}(s))qq_d(s)}d(s) \tag{1}$$

Control effort:

$$u(s)=\frac{q(s)}{1+(p(s)-\tilde{p}(s))qq_d(s)}r(s)+\frac{qq_d(s)p_d(s)}{1+(p(s)-\tilde{p}(s))qq_d(s)}d(s) \tag{2}$$

Comments:

Here $qq_d(s,\lambda)$ can be selected first for good disturbance rejection and then the set point filter $q(s,\lambda_r)$ can be chosen independently for good set point tracking.

4.2 Design for Stable Perfect model

For perfect model, $p(s) = \tilde{p}(s)$; therefore equations (1) and (2) reduce to:

$$y(s) = p(s)q(s)r(s) + \left(1 - p(s)q(s)q_d(s)\right)p_d(s)d(s) \qquad (3)$$

$$u(s) = q(s)r(s) + q(s)q_d(s)p_d(s)d(s) \qquad (4)$$

4.2.1 Design Procedure

Step 1: Select $q(s,\lambda_r)$ as in the 1DF procedure. That is, q inverts a portion of the process model, $\tilde{p}(s)$. Select the controller filter as $1/(\lambda s+1)^r$, where r is the relative order of the inverted part of the model.

Step 2: Select $q_d(s,\lambda)$ as:

$$q_d(s,\lambda) = \frac{\sum_{i=0}^{n} \alpha_i s^i}{(\lambda s + 1)^n}; \qquad \alpha_0 = 1 \qquad 5$$

where n is the number of poles in $p_d(s)$ to be cancelled by the zeros of $\left(1 - p(s)q(s)q_d(s)\right)$.

Step 3: Select a trial value for the filter-time constant λ.

Step 4: Find the value of α_i by solving Equation (6) for each n distinct poles of $p_d(s)$ to be removed:

$$\left(1 - p(s)q(s)q_d(s,\lambda,\alpha)\right)\Big|_{s=-1/\tau_i} = 0; \qquad i = 1,2,\cdots,n \qquad 6$$

if $p_d(s)$ contains repeated pole, then the derivative of equation (6) is set to zero, up to order one less than the number of repeated poles.

Step 5: Refine the value of λ and repeat step 4 untill a desired performance is obtained.

***Example* 4.1**: $p_d(s) = p(s) = \tilde{p}(s) = e^{-s}/(4s+1)$

For step 1, the process model has no zeros and the relative order is one, we design the set point filter as before to be:

$$q(s) = \frac{4s+1}{\lambda s+1} \qquad 7$$

The filter time constant is determined as follows:

$$\lambda \geq \left(\lim_{s\to\infty} \frac{D(s)N(0)}{20s^r N(s)D(0)} \right)^{1/r} \qquad 8$$

$$\lambda \geq \left(\lim_{s\to\infty} \frac{(4s+1)(1)}{20s(1)(1)} \right)^{1} = 0.2 \qquad 9$$

The filter time constant can also be determined from the Bode plot. The bode plot of $q(s)$ with $\lambda = 0.2$ shown in Figure 2 also indicates that at $\omega = 1$, the peak is less than 20 or that the ratio of the final value to the initial value is around 20.

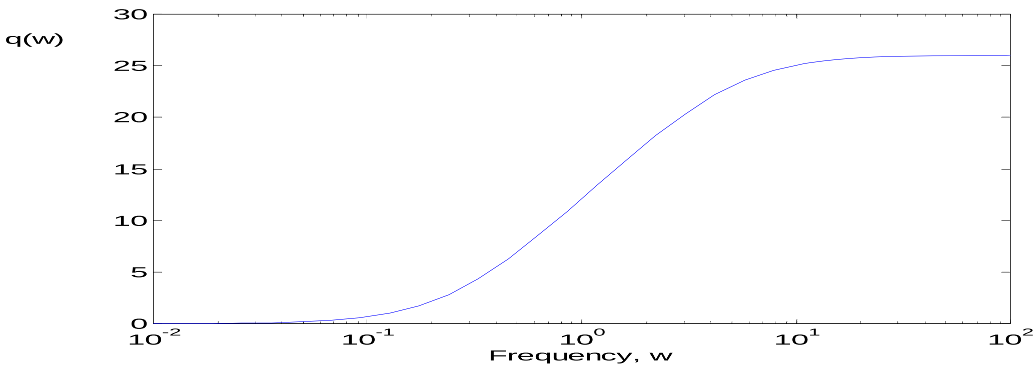


Figure 2: Bode plot for q(s) in Equation (7)

Therefore, the set point filter is:

$$\boxed{q(s) = \frac{4s+1}{0.2s+1}} \qquad 10$$

For step 2, the disturbance dynamics, p_d has one pole (thus, $n = 1$) at $s = -1/4$, we select q_d such that:

$$q_d(s) = \frac{(1+\alpha s)}{(\lambda s + 1)} \tag{11}$$

To select α, we solve the following:

$$(1 - p(s)q(s)q_d(s))\ \Big|_{s=-1/4} = 1 - \frac{e^{-s}}{4s+1}\frac{4s+1}{0.2s+1}\frac{1+\alpha s}{\lambda s+1}\ \Big|_{s=-1/4} = 0 \tag{12}$$

$$\frac{(\lambda s+1)(0.2s+1) - (1+\alpha s)e^{-s}}{(\lambda s+1)(0.2s+1)}\ \Big|_{s=-1/4} = 0 \tag{13}$$

$$(\lambda s+1)(0.2s+1) - (1+\alpha s)e^{-s}\ \Big|_{s=-1/4} = 0 \tag{14}$$

$$(-\lambda/4+1)(-0.2/4+1) - (1-\alpha/4)e^{1/4} = 0 \tag{15}$$

Setting λ = 0.2 gives α = 1.189. Consequently, the controller is:

$$\boxed{q_d(s) = \frac{(1+1.189s)}{(0.2s+1)}} \tag{16}$$

$$\boxed{q(s)q_d(s) = \frac{(1.189s+1)(4s+1)}{(0.2s+1)^2}} \tag{17}$$

Setting λ = 0.59 gives α = 2.736. Consequently, the controller is:

$$\boxed{q_d(s) = \frac{(1+1.736s)}{(0.59s+1)}} \tag{18}$$

$$\boxed{q(s)q_d(s) = \frac{(1.736s+1)(4s+1)}{(0.59s+1)(0.59s+1)}} \tag{19}$$

The closed-loop response for disturbance using 1DF controllers ($q_d(s)$ = 1) is:

$$y(s) = \left(1 - p(s)q(s)\right)p_d(s)d(s) =$$

$$\left(1 - \frac{e^{-s}}{4s+1}\frac{4s+1}{0.2+1}\frac{e^{-s}}{4s+1}\right)d(s) = (1 - \frac{e^{-s}}{0.2s+1})\frac{e^{-s}}{4s+1}d(s) \qquad 20$$

The closed-loop response for disturbance using 2DF controller with α = 0.2 is:

$$y(s) = \left(1 - p(s)qq_d(s)\right)p_d(s)d(s) =$$

$$\left(1 - \frac{e^{-s}}{4s+1}\frac{(1.189s+1)(4s+1)}{(0.2+1)^2}\right)\frac{e^{-s}}{4s+1}d(s) = \qquad 21$$

$$\left(1 - \frac{(1.189s+1)e^{-s}}{(0.2s+1)^2}\right)\frac{e^{-s}}{4s+1}d(s)$$

The closed-loop response for disturbance using 2DF controller with α = 0.59 is:

$$y(s) = \left(1 - p(s)qq_d(s)\right)p_d(s)d(s) =$$

$$\left(1 - \frac{e^{-s}}{4s+1}\frac{(1.736s+1)(4s+1)}{(0.59+1)(0.59s+1)}\right)\frac{e^{-s}}{4s+1}d(s) = \qquad 22$$

$$\left(1 - \frac{(1.736s+1)e^{-s}}{(0.59s+1)^2}\right)\frac{e^{-s}}{4s+1}d(s)$$

Comparison of the three controllers for step change in the disturbance is shown in figure 3.

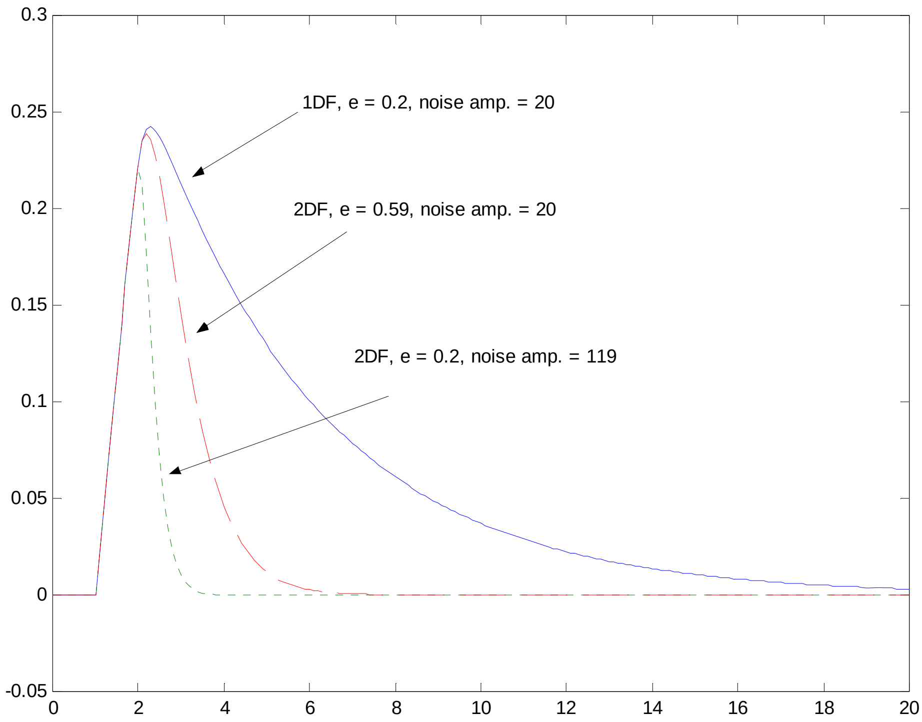


Figure 3: Comparison of the three controllers for step change in disturbance variable.

Note: to obtain the plot in figure 5, use MATLAB commands: tf, set('inputdelay'), y=step(f)-step(p).

Remarks:

- The 2DF controllers outperform the 1DF controller in the sense of faster settling time.
- Comparison of the 2DF controllers indicates that at λ = 0.59 the response has better noise reduction (58 to 119) but at the expense of slower response.

Result analysis:

For 1DF controller

The system response to step disturbance is given by:

$$y(s) = (1 - \frac{e^{-s}}{0.2s+1})\frac{e^{-s}}{4s+1}\frac{1}{s} \qquad 23$$

Applying the inverse of LAPLAC gives:

$$y(t) = 0.274e^{-0.25(t-2)} - 0.0526e^{-5(t-2)}; \qquad t \geq 2 \qquad 24$$

It is obvious form equation (24) that the term $0.274e^{-0.25(t-2)}$ gives rise to the long tail (slow) response in Figure 3.

- To remove this long tail, one could select $q(s)$ such that the term $(1-p(s)q(s))$ has a zero at s = –0.25 to eliminate the pole in the disturbance lag at s = –0.25.
- However, modifying $q(s)$ will adversely affect the system response to set point, recall that in the 1DF structure we are using one controller for both set point and disturbance.

For 2DF controller

- The best way to determine the filter time constant λ and the controller zero, α is to solve simultaneously equation (8) and (12).

***Example* 4.2: Lead Process with two repeated roots**

$$p(s) = p_d(s) = \frac{(2s+1)}{(s+1)^2}e^{-s} \qquad 25$$

The 1DF IMC controller (set point filter) is:

$$q(s) = \frac{(s+1)^2}{(2s+1)(0.025s+1)} \qquad 26$$

With the filter time constant that satisfy the design equation (6) is found to be equal 0.025.

Since the disturbance lag has two poles, i.e. n = 2, then the IMC controller q_d is:

$$q_d(s) = \frac{1+\alpha_1 s+\alpha_s s^2}{(\lambda s+1)^2} \qquad 27$$

Therefore, we need to find the value of α_1 and α_2 that satisfy:

$$\left(1-p(s)qq_d(s)\right)\Big|_{s=-1} = \left|1-\frac{(1+\alpha_1 s+\alpha_2 s^2)}{(\lambda s+1)^3}e^{-s}\right|_{s=-1} = 0 \qquad 28$$

$$\frac{d}{ds}\left(1 - p(s)qq_d(s)\right)\Big|_{s=-1} = \frac{d}{ds}\left|1 - \frac{(1+\alpha_1 s + \alpha_2 s^2)}{(\lambda s + 1)^3} e^{-s}\right|_{s=-1} = 0 \quad 29$$

setting λ = 0.235 in the last two equations, and solving gives:

$$\alpha_2 = 1.35 \quad \alpha_1 = 0.519,$$

$$q_d(s) = \frac{1 + 1.35s + 0.519s^2}{(0.235s+1)^2} \quad 30$$

Therefore, the closed-loop response for disturbance is:

1DF controller:

$$y(s) = \left(1 - p(s)q(s)\right)p_d(s)d(s) = \quad 31$$

$$\left[\frac{2s+1}{(1+s)^2}e^{-s} - \frac{2s+1}{(1+s)^2(1+0.025s)}e^{-2s}\right]d(s)$$

2Df controller:

$$y(s) = \left(1 - p(s)qq_d(s)\right)p_d(s)d(s) = \quad 32$$

$$\left[\frac{2s+1}{(1+s)^2}e^{-s} - \frac{(2s+1)(1+1.35s+0.519s^2)}{(1+s)^2(1+0.325s)^3}e^{-2s}\right]d(s)$$

The output time-response for both controllers for a step change in the disturbance is shown in Figure 4. It is obvious that the 1DF controller provide superior performance over the 2DF controller.

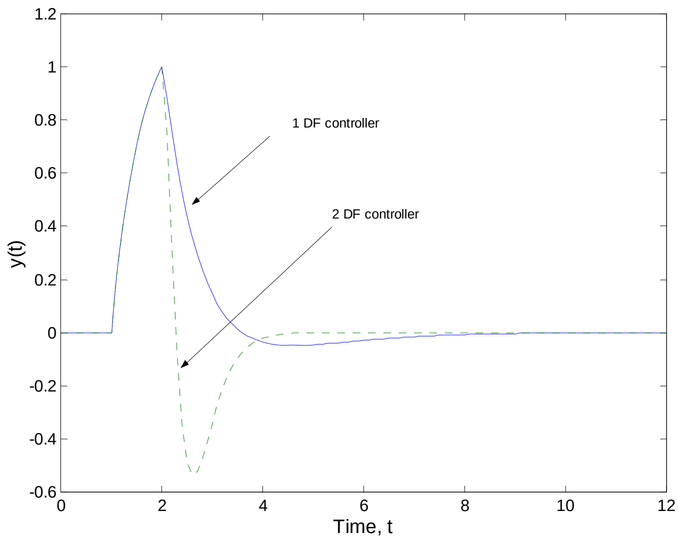


Figure 4: Process response for Example 3.2

The response for control effort for both controllers is given by:

1DF: $$u(s) = -q(s)p_d(s)d(s) = -\frac{e^{-s}}{0.025s+1}d(s)$$

2DF: $$u(s) = -qq_d(s)p_d(s)d(s) = -\frac{(1+1.35s+0.519s^2)}{(0.325s+1)^3}e^{-s}d(s)$$

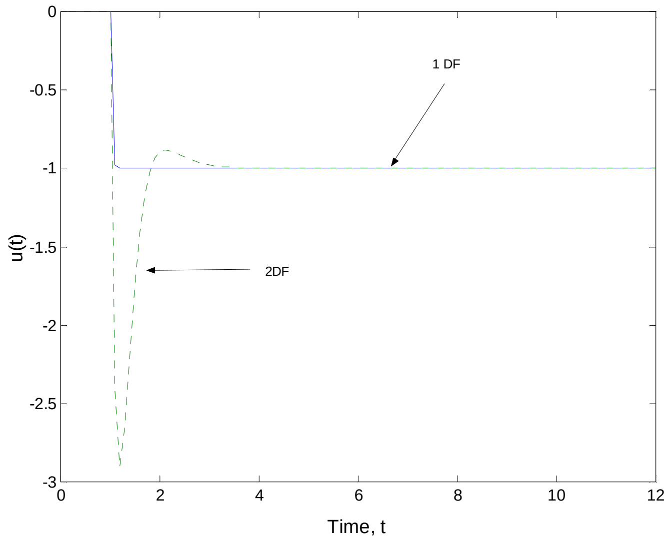


Figure 5: control effort (input) response for example 3.2

A-2 Unstable processes

4.3 Internal Stability

Stable process

Applying ***bounded*** input signal into the process will generate a ***bounded*** output signal.

Unstable process

Applying a ***bounded*** input signal will cause the process output to ***grow*** without bound.

Internal stability

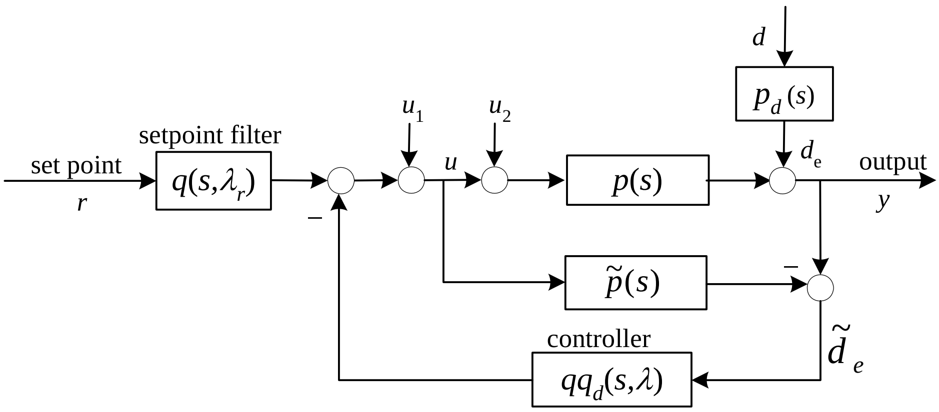


Figure 6: 2DF IMC controller

- *By a definition, a control system is internally stable if bounded input signal injected at any point of the control system, generates a bounded response at any other point.*
- *A linear time invariant control system is internally stable if the transfer function between two points of the block diagram are stable.*

According to the block diagram in Figure 6, there are four inputs and four outputs and the transfer function for a perfect model is given by:

$$\begin{bmatrix} y \\ \bar{y} \\ u \\ \tilde{d}_e \end{bmatrix} = \begin{bmatrix} pq & (1-pqq_d)p_d & p & (1-pqq_d)p \\ pq & pp_d qq_d & p & p^2 qq_d \\ q & p_d qq_d & 1 & pqq_d \\ 0 & p_d & 0 & p \end{bmatrix} \begin{bmatrix} r \\ d \\ u_1 \\ u_2 \end{bmatrix}$$

- If p, p_d, q and q_d are all stable, then all of the transfer functions in equation 32 are stable.
- If p, p_d or q is unstable, then small changes in inputs will cause the outputs to grow without bound.

4.3.1 single-loop implementation of IMC for unstable processes

The 2DF IMC structure can be reconfigured as shown in block diagram of Figure 7.

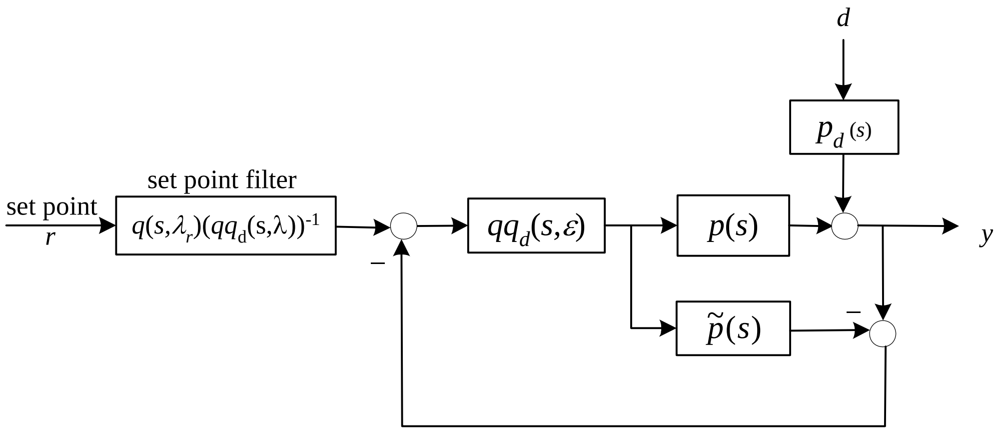


Figure 7: single-loop configuration of a 2DF IMC system

The latest configuration can also be put into the form shown in Figure 8.

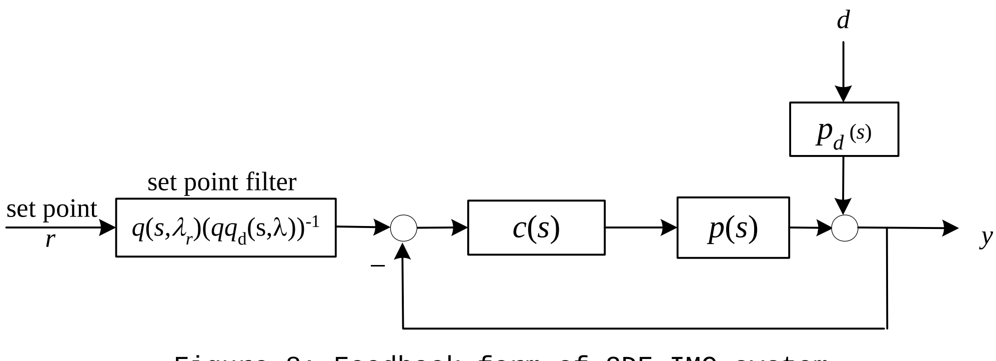


Figure 8: Feedback form of 2DF IMC system

The feedback controller in Figure 8 is given by:

$$c(s) = \frac{qq_d(s,\lambda)}{(1 - p(s)qq_d(s,\lambda))} \qquad 34$$

Lemma (Morari and Zafiriou): The feedback system shown in Figure 8 with $p_d = p$ and no set point filter (i.e. $q(s,\lambda)qq_d^{-}(s,\lambda) = 1$) will be internally stable for a perfect model provided that the terms *pc*/(1+*pc*), *p*/(1+*pc*) and *c*/(1+*pc*) are all stable.

Necessary condition for stability:
The above terms are stable only if (1+*pc*) has no RHP zeros.

Given the expression of $c(s)$ in equation (34), the necessary condition indicates (Morari and Zafiriou):

1. qq_d must be stable.
2. pqq_d must be stable. This requires that qq_d cancels the *unstable* poles of *p*.
3. $(1-pqq_d)p$ must be stable. This requires that the zeros of $(1-pqq_d)p$ cancels the *unstable* poles of *p*.

Note: Satisfying the conditions 1-3 does not necessarily lead to a stable controller, $c(s)$. The reason is that the term $(1-pqq_d)$ my itself has RHP zeros other than those to be cancelled in $p(s)$.

The RHP zero of the term $(1-pqq_d)$ can be avoided by increasing the filter time constant, λ. However, this will degrade the control system performance.

Example* 4.3*:

$$p(s) = p_d(s) = \frac{e^{-s}}{(1-s)} \quad 35$$

Using the same design procedure we have:

$$q_d(s) = \frac{5.1162s+1}{0.5s+1} \quad ; q(s) = \frac{1-s}{0.5s+1}$$

The closed-loop response for disturbance we have:

$$y(s) = \left(1 - p(s)qq_d(s)\right)p_d(s)d(s) \quad 36$$

we note that:

$$qq_d(s) = \frac{(1-s)(5.12s+1)}{(0.5s+1)^2} \quad (37)$$

is stable and:

$$p(s)qq_d(s) = \frac{(5.12s+1)e^{-s}}{(0.5s+1)^2} \quad (38)$$

is also stable because it eliminates the unstable pole of $p(s)$, but

$$\left(1-p(s)qq_d(s)\right)p_d(s)d(s) = \left(1-\frac{(5.12s+1)e^{-s}}{(0.5s+1)^2}\right)\frac{e^{-s}}{1-s}d(s) \quad (39)$$

seems to be unstable because of the unstable pole $s = 1$.

Brosilow design the controller in a single transfer function by approximating the time delay by five-order Pade series:

For $\lambda = 0.5$:

$$c(s) = \frac{qq_d(s)}{1-p(s)qq_d(s)} =$$

$$\frac{-0.32(0.01138s^2+0.128s+1)(0.0175s^2+0.235s+1)0.137s+1)(0.512s+1)}{(0.00497s^2-0.00993s+1)(0.0318s^2-0.0684s+1)(0.0168s+1)s} \quad (40)$$

This controller is unstable, therefore, it is suggested to increase the filter-time constant to provide stability:

For $\lambda = 2.8$:

$$c(s) = \frac{qq_d(s)}{1-p(s)qq_d(s)} =$$

$$\frac{-0.0316(0.0138s^2+0.128s+1)(0.0175s^2+0.235s+1)0.137s+1)(38.25s+1)}{(0.00618s^2+0.0283s+1)(0.0424s^2+4.66e^{-4}s+1)(0.0312s+1)s} \quad (41)$$

Controller s stable, and the overall close-loop system is thus given by:

$$y(s) = \frac{p_d(s)}{1+p(s)c(s)}d(s) \quad (42)$$

Based on equation (41) and the two controllers in equation (39) and (40) the feedback response is shown in Figure 8. Although both responses are stable, it is nit true for long simulation time as shown in figure 9.

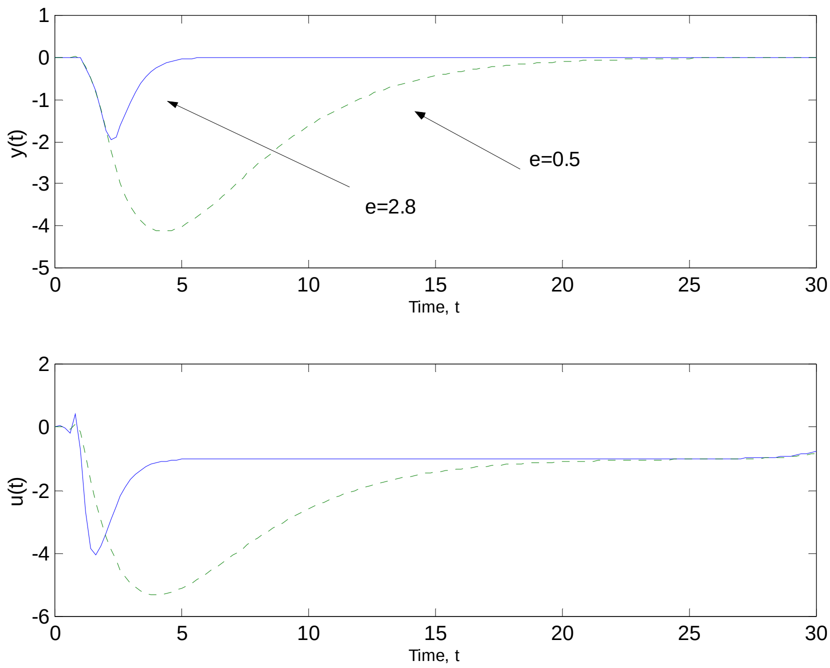


Figure 9: Feedback response for unstable system

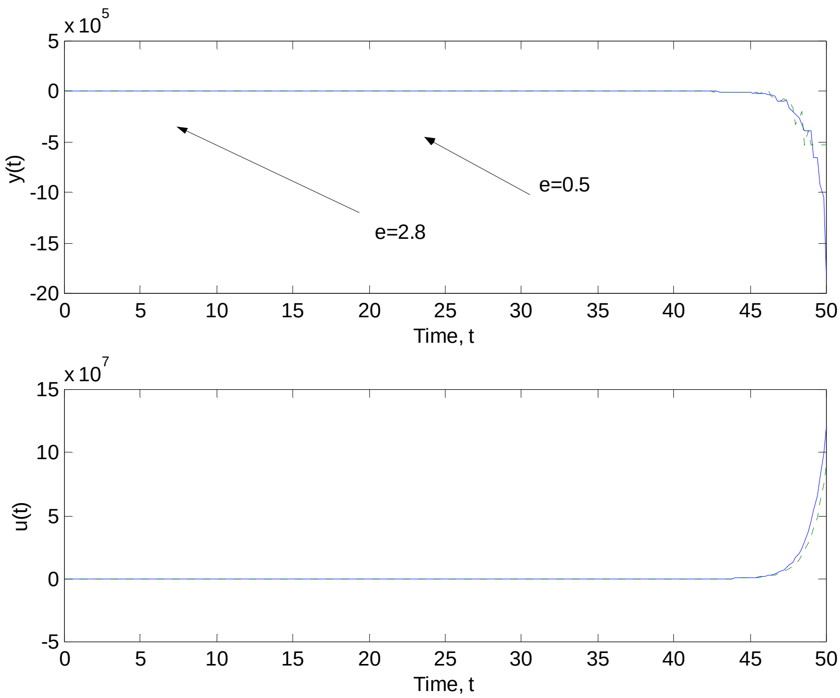
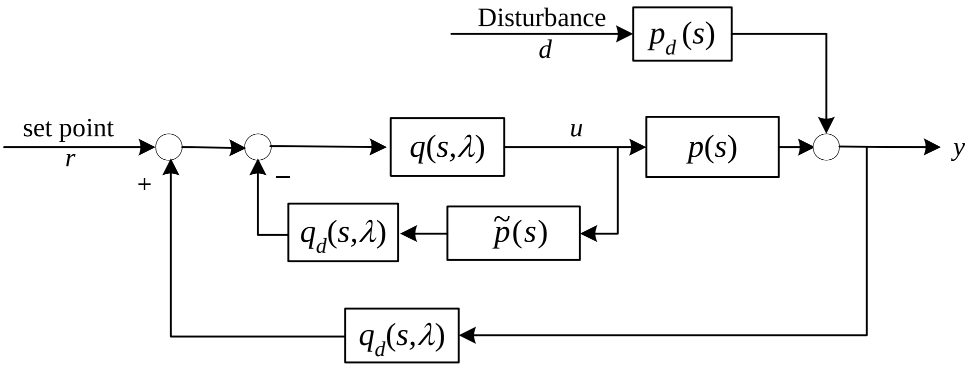


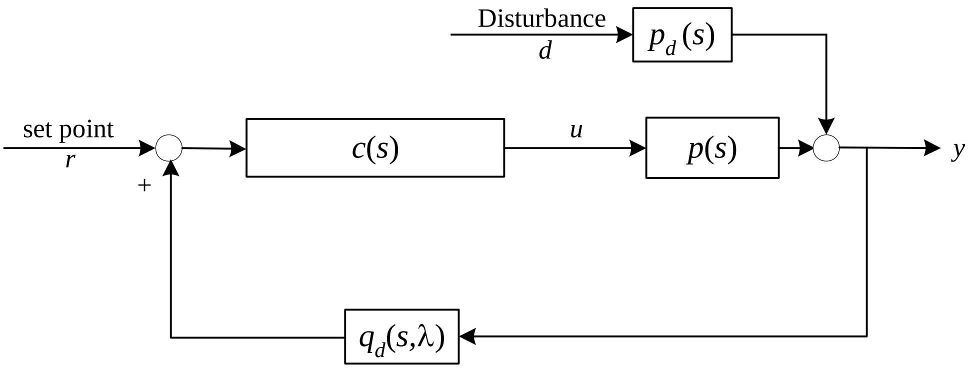
Figure 10: Example 3.3

APPENDIX

The original 2DF structure of IMC



Another representation of the 2DF IMC structure



where $c(s) = \dfrac{u(s)}{e(s)} = \dfrac{q(s)}{1 - \tilde{p}(s)q(s)q_d(s)}$

Therefore according to the last block diagram, one can write:

$$\frac{u(s)}{r(s)} = \frac{c(s)}{1 + p(s)c(s)q_d(s)}$$

$$\frac{u(s)}{d(s)} = \frac{c(s)q_d(s)p_d(s)}{1 + p(s)c(s)q_d(s)}$$

$$\frac{y(s)}{r(s)} = \frac{p(s)c(s)}{1 + p(s)c(s)q_d(s)}$$

$$\frac{y(s)}{d(s)} = \frac{p_d(s)}{1 + p(s)c(s)q_d(s)}$$

Inserting the definition of $c(s)$ into the last equations give:

$$y(s) = \frac{p(s)q(s)}{1 + (p(s) - \tilde{p}(s))q(s)q_d(s)} r(s) + \frac{\left(1 - p(s)q(s)q_d(s)\right)p_d(s)}{1 + (p(s) - \tilde{p}(s))q(s)q_d(s)} d(s)$$

$$u(s) = \frac{q(s)}{1 + (p(s) - \tilde{p}(s))q(s)q_d(s)} r(s) + \frac{q(s)q_d(s)p_d(s)}{1 + (p(s) - \tilde{p}(s))q(s)q_d(s)} d(s)$$