CHAPTER 4

Two-Degree of Freedom Internal Model Control

Objectives of the Chapter

- Introduce the two-degree of freedom (2DF) IMC system and elucidate its advantages over 1DF IMC when step disturbances enter through process lags.
- Provide design methods for 2DF IMC systems with perfect models that give the best possible performance consistent with noisy measurements for inherently stable and inherently unstable processes.

Prerequisite Reading

Chapter 3, "One-Degree of Freedom Internal Model Control"

4.1 INTRODUCTION

The IMC design methods presented of Chapter 3 assume that step disturbances enter into the process output without passing through the process (i.e., $p_d(s) = 1$) in Figure 3.1, which is reproduced in Figure 4.1.

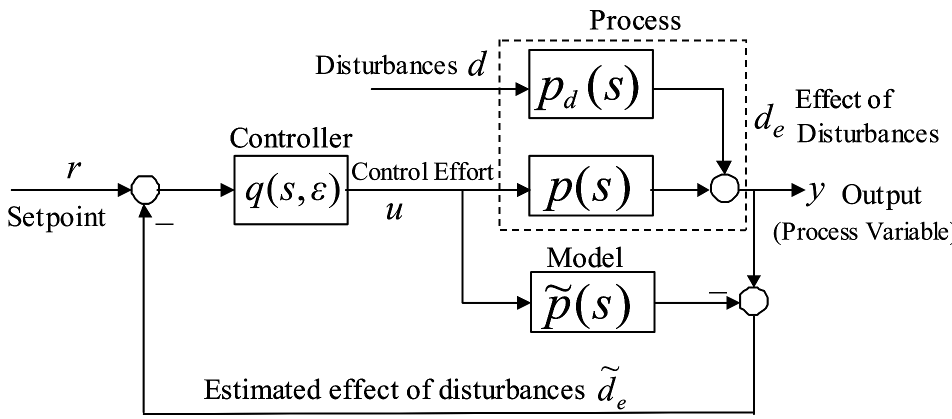

Figure 4.1 The 1DF IMC block diagram (from Chapter 3).

Assuming that the disturbance enters directly into the output allows the controller design to focus on achieving a good response to a step setpoint change. Such a controller equally well suppresses a step disturbance because the signal that enters the controller is the setpoint minus the disturbance estimate. Therefore, as far as the controller is concerned, a positive step disturbance is the same as a negative step setpoint change and vice versa. However, when the disturbance enters through the process, the disturbance transfer function is not one, and a controller designed to suppress a step disturbance will apply an inadequate control effort. The resulting response is often much more sluggish than is desirable. Consider Example 4.1, where the disturbance enters the process in the same way as the control effort so that the disturbance and the process transfer functions are the same.

Example 4.1 1DF Response to Process Disturbance

Process and models are

$$\tilde{p}_d(s) = p_d(s) = \tilde{p}(s) = p(s) = e^{-s}/(4s+1) \qquad (4.1a)$$

The single-degree of freedom IMC controller is (see Chapter 3).

$$q(s) = (4s+1)/(0.2s+1). \qquad (4.1b)$$

(0.2 is the filter-time constant for a noise amplification of 20.)

The resulting control effort, $m(s)$, and output, $y(s)$, for a step disturbance are:

$$m(s) = -\tilde{p}_d(s)q(s)/s \quad = \quad -e^{-s}/s(0.2s+1) \tag{4.1c}$$

$$y(s) = (1-\tilde{p}(s)q(s))\tilde{p}_d(s)/s \quad = \quad \left(1-\frac{e^{-s}}{(0.2s+1)}\right)\frac{e^{-s}}{(4s+1)s}. \tag{4.1d}$$

Figure 4.2 shows the time responses of the output and control effort of equations (4.1c) and (4.1d). The long tail on the response of the output $y(t)$ can be ascribed to the fact that the control effort reaches steady state in about one time unit, while the output, delayed by 1 time unit, has not yet stopped rising. The output response from about 2.4 time units onward is similar to that of an unforced process, starting with an initial condition of about 0.24 and moving to a steady state of zero.

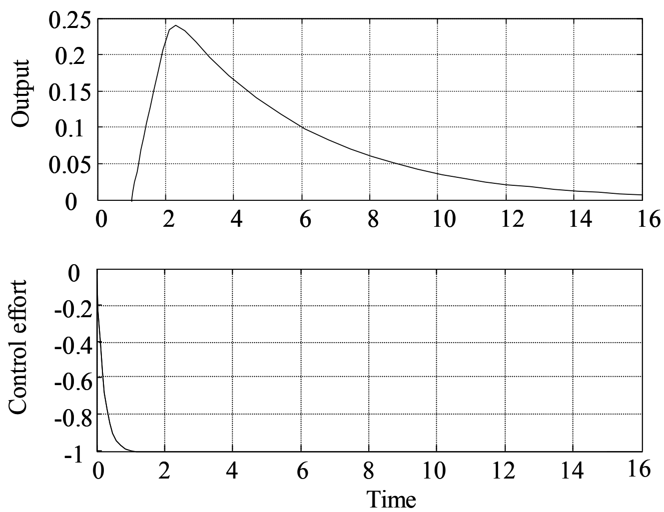

Figure 4.2 Response of 1DF IMC to a disturbance that enters through the process.

Another perspective on the output response can be obtained from the following partial fraction expansion of Eq. (4.1d).

$$y(s) = \left(\frac{1}{s} - \frac{4}{4s+1}\right)e^{-s} - \left(\frac{1}{s} + \frac{.010s}{.2s+1} - \frac{4.21}{4s+1}\right)e^{-2s} \tag{4.1e}$$

Inverting the above and collecting terms for $t \geq 2$ gives

$$y(t) = 0.274e^{-.25(t-2)} - 0.0526e^{-5(t-2)};\ t \geq 2 \tag{4.1f}$$

The term $0.274e^{-.25(t-2)}$ gives rise to the long tail in the response shown in Figure 4.2. To remove this term, we can choose the IMC controller so that the term $(1-\tilde{p}(s)q(s))$ has a zero at $s = -.25$, thereby canceling the pole in the disturbance lag at $s = -.25$. However, choosing the IMC controller so that the term $(1-\tilde{p}(s)q(s))$ has a zero at the pole of the disturbance lag will generally seriously degrade the setpoint response. (Recall that the setpoint to output transfer function is $\tilde{p}(s)q(s)$.) To avoid degrading the setpoint response in favor of the disturbance response, we use the two-degree of freedom control structure of Figure 4.3.

4.2 THE STRUCTURE OF 2DF IMC

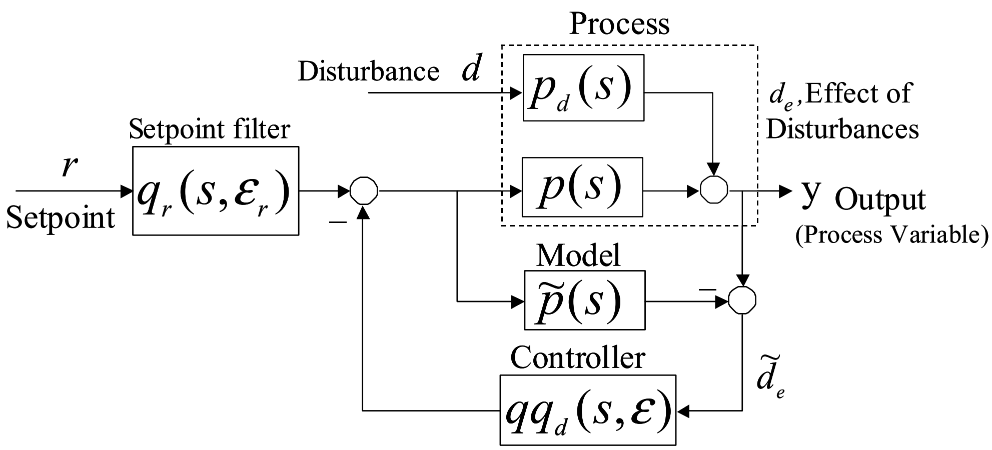

Figure 4.3 2DF IMC Structure.

The controller $qq_d(s,\varepsilon)$ in Figure 4.3 is designed to reject disturbances while the setpoint controller $q(s,\varepsilon)$ is designed to shape the response to setpoint changes. Henceforth, we will refer to the setpoint controller as the setpoint filter in order to be consistent with industrial terminology.

The perfect model output and control effort responses for Figure 4.3 are

$$y(s) = \tilde{p}(s)q(s,\varepsilon_r)r(s) + (1-\tilde{p}(s)qq_d(s,\varepsilon))p_d(s)d(s) \qquad (4.2a)$$

$$m(s) = q(s,\varepsilon_r)r(s) + qq_d(s,\varepsilon)p_d(s)d(s). \qquad (4.2b)$$

Returning to Example 4.1, Figure 4.4 shows the response of the control system of Figure 4.3 to a step disturbance with q_d chosen as

$$qq_d(s) = (4s+1)(1.19s+1)/(.2s+1)^2, \qquad (4.2c)$$

the filter-time constant in Eq. (4.2c) is 0.2 (i.e., $\varepsilon = 0.2$). There is a substantial improvement in the speed with which the disturbance on the output is eliminated. Also note, however, that the improved output response requires a substantially more aggressive control effort.

The next section shows how Eq. (4.2c) was obtained, and presents design procedures for both $qq_d(s,\varepsilon)$ and $q_r(s,\varepsilon_r)$ for a perfect model. Tuning both $qq_d(s,\varepsilon)$ and $q_r(s,\varepsilon_r)$ to accommodate model uncertainty is discussed in Chapter 7.

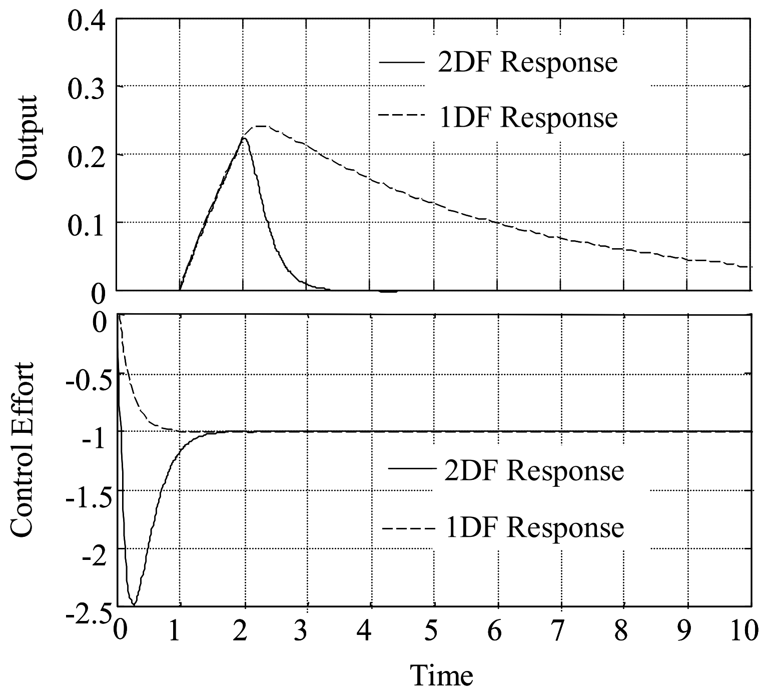

Figure 4.4 Comparison of 1DF and 2DF IMC responses to a step disturbance to the process of Example 4.1, $qq_d(s,.2) = (4s+1)(1.19s+1)/(.2s+1)^2$.

4.3 DESIGN FOR STABLE PROCESSES

4.3.1 Design of the Setpoint Filter, $q(s,\varepsilon_r)$

The setpoint filter $q_r(s,\varepsilon_r)$ in Figure 4.3 is designed like a single-degree of freedom controller using the methods in Chapter 3. However, since there is generally no noise on the setpoint, there is no noise amplification limit on ε_r. Nonetheless, very small values of ε_r are not recommended because of the likelihood of control effort saturation. Unless a model state feedback implementation is used (see Chapter 5), smaller filter-time constants can actually lead to slower output responses because of control effort saturation.

4.3.2 Design of the Feedback Controller, $qq_d(s,\varepsilon)$

The transfer function between output and disturbance for Figure 4.3 for a perfect model is

$$y(s) = (1-\widetilde{p}(s)qq_d(s,\varepsilon))\widetilde{p}_d(s)d(s). \tag{4.3}$$

To design $qq_d(s,\varepsilon)$ for a perfect model, it is convenient to consider $qq_d(s,\varepsilon)$ to be composed of two terms, $q(s,\varepsilon)$ and $q_d(s,\varepsilon)$. The design then proceeds as follows:

1) Select $q(s,\varepsilon)$ as in Chapter 3. That is, $q(s,\varepsilon)$ inverts a portion of the process model $\widetilde{p}(s)$. Select the controller filter as $1/(\varepsilon s+1)^r$, where r is the relative order of the part of the process model that is inverted by $q(s,\varepsilon)$.

2) Select $q_d(s,\varepsilon)$ as

$$q_d(s,\varepsilon,\alpha) = \frac{\sum_{i=0}^{n}\alpha_i s^i}{(\varepsilon s+1)^n}; \quad \alpha_0 \equiv 1, \tag{4.4}$$

where n is the number of poles in $\widetilde{p}_d(s)$ to be cancelled by the zeros of $(1-\widetilde{p}(s)qq_d(s))$.

3) Select a trial value for the filter-time constant ε.

4) Find the values of α_i by solving Eq. (4.5) for each of the n distinct poles of $\widetilde{p}_d(s)$ that are to be removed from the disturbance response.

$$(1-\widetilde{p}(s)qq_d(s,\varepsilon,\alpha))\Big|_{s=-1/\tau_i} = 0; \quad i = 1, 2, \dots n, \tag{4.5}$$

where τ_i is the time constant associated with the i^{th} pole of $\widetilde{p}_d(s)$.

If any of the poles occur in complex conjugate pairs, then both the real and imaginary parts of equation are set to zero for one of the complex conjugate pairs. The set of equations given by Eq. (4.5) are linear in the parameters α_i, no matter whether any of the poles of $\widetilde{p}_d(s)$ are real or complex.

If $\widetilde{p}_d(s)$ contains repeated poles (e.g., $\widetilde{p}_d(s) = 1/(\tau_j s+1)^r$), then the derivatives of Eq. (4.5) are set to zero, up to order one less than the number of repeated poles. For example, if $\widetilde{p}_d(s) = 1/(\tau_j s+1)^r$ then we solve

$$(1-\widetilde{p}(s)qq_d(s,\varepsilon,\alpha))\Big|_{s=-1/\tau_j} = 0 \tag{4.6a}$$

$$\frac{d^k}{ds^k}(\widetilde{p}(s)qq_d(s,\varepsilon,\alpha)\,|_{s=-1/\tau_j} = 0; \quad k = 1, 2,....\, r-1 \tag{4.6b}$$

5) Adjust the value for ε, and repeat step 4 until the desired noise amplification is achieved. A few trials are usually sufficient to achieve a noise amplification factor close enough to the desired value. Of course, one could solve simultaneously for the α_i that satisfy step 4 and the desired noise amplification. However, solving simultaneously for α_i and ε requires the solution of a set of nonlinear equations.

The software IMCTUNE automatically provides values for all α_i for up to fourth order disturbance lags in $\widetilde{p}_d(s)$ given a value for the filter-time constant ε.

To illustrate the procedure, we consider again the process of Example 4.1.

Example 4.2 Solving for the feedback controller numerator coefficient(s) α_i

In step 1, we select the setpoint filter $q(s)$ to invert the invertible portion of the process model $\widetilde{p}(s)$.

$$\widetilde{p}_d(s) = p_d(s) = \widetilde{p}(s) = p(s) = e^{-s}\big/(4s+1) \tag{4.7a}$$

$$q(s) = (4s+1)/(.2s+1) \tag{4.7b}$$

In step 2 we design q_d(s) so that the zeros of $(1-\widetilde{p}(s)qq_d(s))$ cancel the poles of $p_d(s)$. Since $p_d(s)$ has a single pole at -1/4, we select $q_d(s)$ as

$$q_d(s) = \frac{(\alpha s+1)}{(\varepsilon s+1)}. \tag{4.7c}$$

The constant α is chosen so that $(1-\widetilde{p}(s)qq_d(s))$ has a zero at $s = -$ ¼. That is,

$$(1-\widetilde{p}(s)qq_d(s,\varepsilon,\alpha))\Big|_{s=-1/4} = \left(\frac{1-(-\alpha/4+1)e^{1/4}}{(\varepsilon s+1)}\right) = 0 \tag{4.7d}$$

Selecting $\varepsilon = 0.2$ in Eq. (4.7d) gives $\alpha = 1.189$. Expanding Eq. (4.3) in partial fractions with $\alpha = 1.189$ for a unit step disturbance gives

$$y(s) = \left(\frac{1}{s} - \frac{1}{s+.25}\right)e^{-s} - \left(\frac{1}{s} - \frac{1.301}{(s+.5)^2} - \frac{.2213}{(s+5)} - \frac{.7787}{s+.25}\right)e^{-2s}. \tag{4.7e}$$

Inverting Eq. (4.7e) and collecting terms for $t \geq 2$ gives

$$y(t) = (1.39(t-2) - .221)e^{-s(t-2)} - 2\times10^{-4}e^{-.25(t-2)} \tag{4.7f}$$

The coefficient of the term $e^{-.25(t-2)}$ can be made as small as we like by carrying more significant figures in α. The graph of $y(t)$ given by Eq. (4.7f) is the solid line in Figure 4.4 labeled as "2DF Response."

In the foregoing, we chose the filter-time constant ε as 0.2 because this was the value that yields a noise amplification factor of 20 for the IMC controller $q(s)$. However, the noise amplification factor that is really of interest is that of $qq_d(s)$, and this is given by the maximum value of $|qq_d(j,\omega)/qq_d(0)|$ over all frequencies, ω. Usually this maximum occurs at $\omega = \infty$. For $\varepsilon = .2$ and $\alpha = 1.189$, the noise amplification factor is 119. Thus, this filter-time constant is much too small. Solving for a filter-time constant, ε, and α that satisfies the noise amplification factor and satisfies Eq. (4.7d), gives $\varepsilon = .59$ and $\alpha = 1.736$. The response of the process output to a step disturbance entering through $p_d(s)$ is shown in Figure 4.5. Clearly, insisting on not amplifying noise by more than a factor of 20 has lost some of the advantage of the two-degree of freedom control system. However, a settling time of 6 time units is still much better than a settling time of 20 time units.

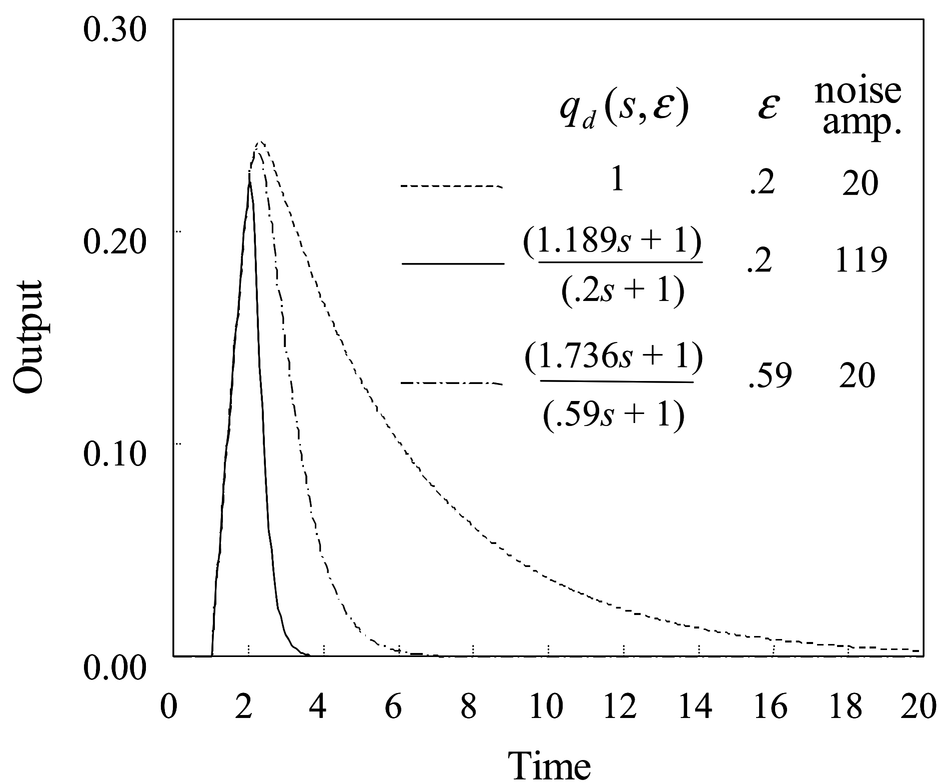

Figure 4.5 Comparison of 1DF and 2DF IMC responses to a step disturbance to the process of Example 4.2, $q(s)=(4s+1)/(\varepsilon s+1)$.

Example 4.3 A Lead Process

This example demonstrates that for a lead process (one whose frequency response initially increases before eventually decreasing), there is no advantage to using a 2DF over 1DF control system.

The process and disturbance lag are

$$p_d(s) = p(s) = \frac{(2s+1)e^{-s}}{(s+1)^2}. \tag{4.8a}$$

The one-degree of freedom IMC controller for the above process is

$$q(s) = \frac{(s+1)^2}{(2s+1)(.025s+1)}. \tag{4.8b}$$

A two-degree of freedom controller to cancel the linear term of the disturbance lag (i.e., $2s + 1$) is

$$qq_d(s) = \frac{(s+1)^2(0.969s+1)}{(2s+1)(0.156s+1)^2}. \tag{4.8c}$$

A two-degree of freedom controller to cancel both disturbance lags (i.e., $(s + 1)^2$) is

$$qq_d(s) = \frac{(s+1)^2(0.519s^2+1.35s+1)}{(2s+1)(0.235s+1)^3}. \tag{4.8d}$$

Figure 4.6 gives the time responses to a step disturbance for each of the controllers.

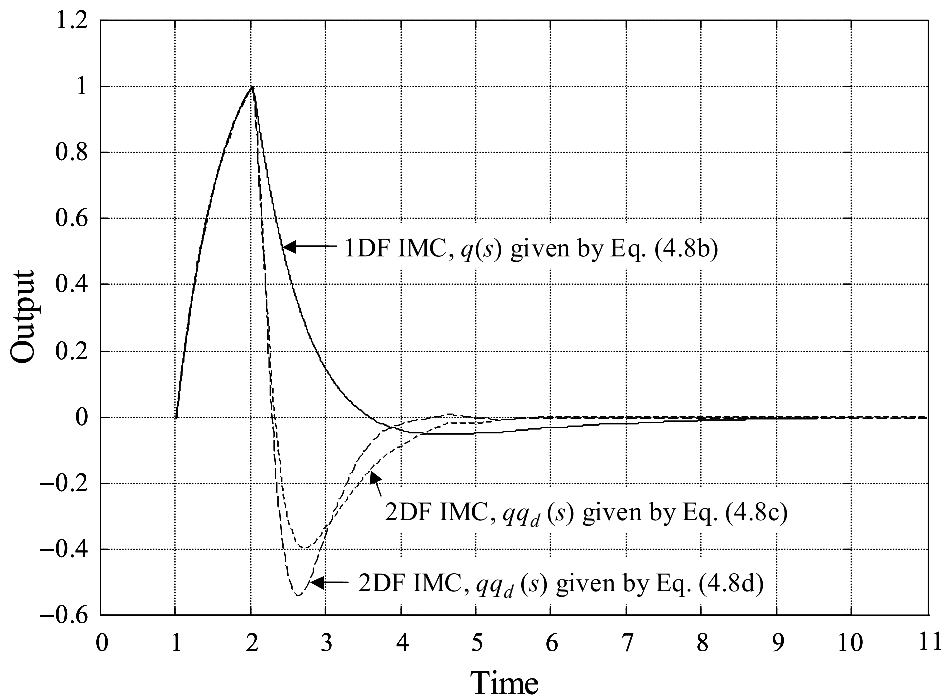

Figure 4.6 Comparison of 1DF and 2DF IMC responses to a step disturbance to the process of Example 4.3.

Clearly, for this example, the one-degree of freedom controller gives superior performance.

Example 4.4 An Underdamped Process

The purpose of this example is to demonstrate the advantage of a two-degree of freedom control system over a single-degree of freedom control system when the disturbance passes through an underdamped process (i.e., a process with complex poles). We also demonstrate the computation of the q_d part of the feedback controller for such a process.

The process and disturbance lag are

$$p_d(s) = p(s) = e^{-s} / (s^2 + .2s + 1). \tag{4.9a}$$

The 1DF controller for the above process that has a noise amplification factor of 20 is

$$q(s) = \frac{(s^2 + .2s + 1)}{(.22s + 1)^2}. \tag{4.9b}$$

The two-degree of freedom controller for the above process that has a noise amplification factor of 20 is

$$qq_d(s) = \frac{(s^2 + .2s + 1)(2.4s^2 + .32s + 1)}{(.6s + 1)^4}. \tag{4.9c}$$

Figure 4.7 shows the responses of the single and two-degree of freedom control systems for Example 4 for a step disturbance. The response of the two-degree of freedom control system is clearly much superior to that of the single-degree of freedom system.

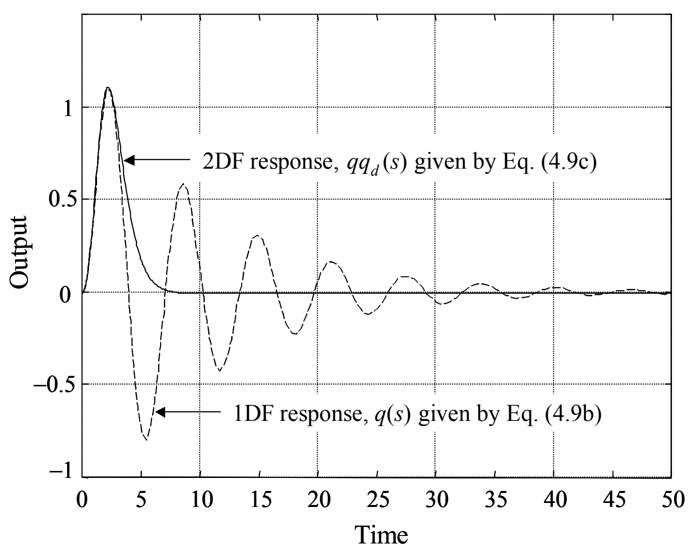

Figure 4.7 1DF and 2DF IMC responses to a step disturbance for Example 4.4.

4.4 DESIGN FOR UNSTABLE PROCESSES

4.4.1 Internal Stability

When the process is unstable, both 1DF and 2DF IMC systems are internally unstable. That is, applying only bounded inputs will cause one or more signals in the block diagram (see figures 4.1 and 4.3) to grow without bound no matter how the controllers $q(s)$ and $qq_d(s)$ are chosen. To show that the IMC structures are internally unstable, we follow the approach in Morari and Zafiriou (1989).

By definition, a control system is internally stable if bounded inputs, injected at any point of the control system, generate bounded responses at any other point. A linear time invariant control system is internally stable if the transfer functions between any two points of the block diagram are stable. Figure 4.8 is the same as Figure 4.3, except for the addition of the two inputs u_1 and u_2. The inputs are the setpoint r the disturbance d and two error, or noise, inputs u_1 and u_2.

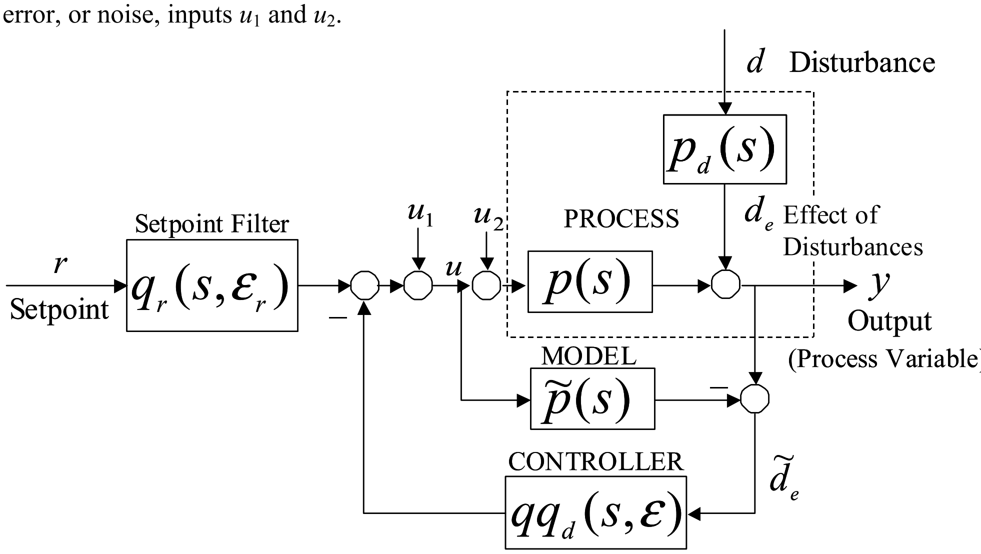

Figure 4.8 Block diagram of 2DF IMC system with additional inputs (u_1 and u_2) for deriving internal stability conditions.

For the analysis of internal stability, it is sufficient to take the control system outputs as the process outputs y, the model output $\tilde{y}$, the control effort u, and the estimate of the effect of disturbances on the process output $\tilde{d}_e$. The input and output transfer functions for a perfect model are

$$\begin{bmatrix} y \\ \tilde{y} \\ u \\ \tilde{d}_e \end{bmatrix} = \begin{bmatrix} pq & (1-pqq_d)p_d & p & (1-pqq_d)p \\ pq & pp_d qq_d & p & p^2 qq_d \\ q & p_d qq_d & 1 & pqq_d \\ 0 & p_d & 0 & p \end{bmatrix} \begin{bmatrix} r \\ d \\ u_1 \\ u_2 \end{bmatrix}. \tag{4.10}$$

If p, p_d, q, and q_d are all stable, then all of the transfer functions in Eq. (4.10) are stable. However, if p, p_d , or q is unstable, then small changes in the inputs r, d, u_1, and u_2 will cause the outputs, y, $\tilde{y}$, u, and $\tilde{d}_e$ to grow without bound. Thus, for unstable processes, 1DF or 2DF control systems should not be implemented as shown in figures 4.1 or 4.3, no matter how q and q_d are selected. All is not lost, however. An internally stable 2DF control system for unstable processes obtained from an IMC design can be implemented in the form of a single-loop feedback control system as discussed below.

4.4.2 Single-loop Implementation of IMC for Unstable Processes

Just as was done for 1DF control systems in Chapter 3 (see Figure 3.2), the two-degree of freedom control system of Figure 4.3 can be collapsed into a single-loop feedback control system. First, Figure 4.3 is reconfigured into Figure 4.9 by moving the controller $qq_d(s,\varepsilon)$ out of the feedback path.

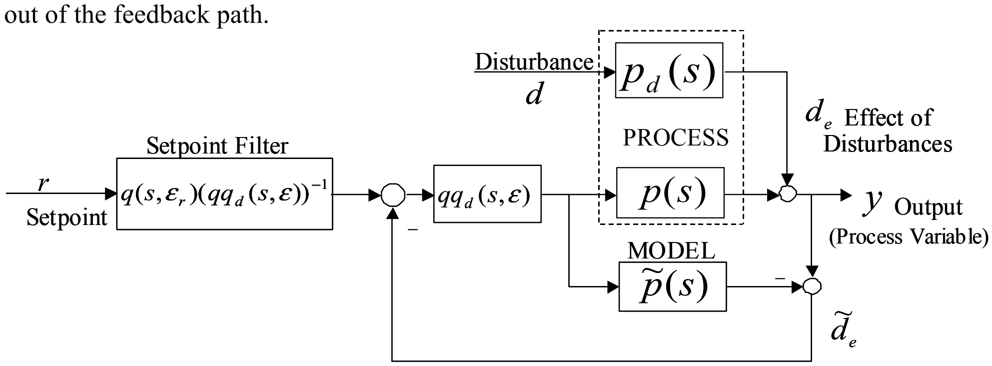

Figure 4.9 Single-loop configuration of a 2DF IMC system.

Next, the feedback loop around the model is collapsed to obtain the Figure 4.10.

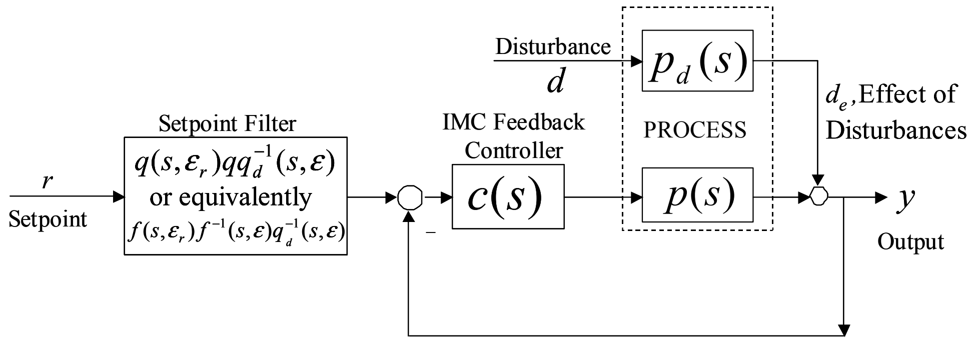

Figure 4.10 Feedback form of 2DF IMC system.

The feedback controller $c(s)$ in Figure 4.10 is

$$c(s) = \frac{qq_d(s,\varepsilon)}{(1-\tilde{p}(s)qq_d(s,\varepsilon))}. \tag{4.11a}$$

The setpoint filter of Figure 4.9 is converted into the setpoint filter of Figure 4.10 using the relationship

$$q(s,\varepsilon_r)q^{-1}(s,\varepsilon) = f(s,\varepsilon_r)f^{-1}(s,\varepsilon), \tag{4.11b}$$

where $f(s,\varepsilon) \equiv 1/(\varepsilon s+1)^r$.

Morari and Zafiriou (1989) show that the feedback system given in Figure 4.10 with $p_d = p$ and no setpoint filter (i.e., $q(s,\varepsilon_r)qq_d^{-1}(s,\varepsilon)=1$) will be internally stable for a perfect model provided that the terms $\tilde{p}(s)c(s)/(1+\tilde{p}(s)c(s))$, $\tilde{p}(s)/(1+\tilde{p}(s)c(s))$, and $c(s)/(1+\tilde{p}(s)c(s))$ are all stable. A necessary condition for the foregoing terms to be stable is that the term $(1+\tilde{p}(s)c(s))$ has no right half plane zeros.

Substituting the controller *c(s)*, given by Eq. (4.11a), into the conditions given above places the following conditions on the design of the IMC controllers (Morari and Zafiriou, 1989).

1) $qq_d(s,\varepsilon)$ must be stable.

2) $\tilde{p}(s)qq_d(s,\varepsilon)$ must be stable. This requires that the zeros of $qq_d(s,\varepsilon)$ cancel the unstable poles of $\tilde{p}(s)$.

3) $(1-\tilde{p}(s)qq_d(s,\varepsilon))\tilde{p}(s)$ must be stable. This requires that the zeros of $(1-\tilde{p}(s)qq_d(s,\varepsilon))$ also cancel the unstable poles of $\tilde{p}(s)$.

In addition, in order for Figure 4.10, as drawn, to be internally stable, $q_d^{-1}(s,\varepsilon)$ must be stable, and the unstable poles of $p_d(s)$ must be *exactly* the same as the poles of $p(s)$ (i.e., any unstable poles in $p_d(s)$ must arise because the disturbance passes through an unstable part of the process in addition to any other stable lags that the disturbance may pass through).

It is important to note that satisfying conditions 1 through 3, does not necessarily lead to a stable controller, *c(s)*, as given by Eq. (4.11a). The problem is that the term $(1-\tilde{p}(s)qq_d(s,\varepsilon))$ can have right half plane zeros in addition to those needed to cancel the right half plane poles of $\tilde{p}(s)$. While it is possible to have a stable closed-loop control system with an unstable controller and unstable process, it is generally not advisable to implement such a control system. The necessary conversion of the controller given by Eq. (4.11a) into a single transfer function usually requires a Padé approximation of a dead time. When the controller is unstable, this approximation can lead to closed-loop instability. As we shall see in the example that follows, for stable controllers $qq_d(s,\varepsilon)$ it is usually possible to avoid $(1-\tilde{p}(s)qq_d(s,\varepsilon))$ having additional, undesired right half plane zeros by increasing the filter-time constant ε.

Since increasing the filter-time constant beyond that required to satisfy maximum noise amplification specifications or to accommodate process uncertainty degrades control system performance, Brosilow and Cheng, (1987) and Berber and Brosilow, (1999) developed an internally stable method of implementing 2DF IMC systems. A discussion of their method is, however, beyond the scope of this text.

The following example demonstrates the design of a 2DF controller for an unstable process and illustrates how the behavior of unstable processes under feedback control differs from that of stable processes.

Example 4.5 Control of a FOPDT Unstable Process

The process and model are

$$p(s)=\tilde{p}(s)=\frac{e^{-s}}{(-s+1)}. \tag{4.12}$$

Following the procedure outlined in Section 4.3.2 and selecting a filter-time constant for a maximum noise amplification of approximately 20 yields the following IMC controllers:

$$q(s)=\frac{(-s+1)}{(0.5s+1)};\quad q_d(s)=\frac{(5.1162s+1)}{(0.5s+1)}. \tag{4.13}$$

The ideal disturbance response of the control system of Figure 4.9 is obtained from Eq. (4.3), which is repeated below.

$$y(s)=(1-\tilde{p}(s)qq_d(s,\varepsilon))\tilde{p}(s)d(s), \tag{4.14}$$

where y(s) is the process output and $d(s)$ is a step disturbance (i.e. $d(s) = 1/s$). Using a five over five Padé approximation for the dead time of one in $\widetilde{p}(s)$, and being careful to cancel the common factors in Eq. (4.14) yields

$$y(s) = \frac{-3.12(0.00168s+1)(0.00497s^2-0.00993s+1)(0.0318s^2-0.0684s+1)}{(0.0138s^2+0.128s+1)(0.0175s^2+0.235s+1)(0.137s+1)(0.5s+1)^2}. \tag{4.15}$$

The quadratic terms in Eq. (4.15) with complex roots arise from the Padé approximation of the dead time. Notice that $y(s)$ is stable. The time response of Eq. (4.15) is given in Figure 4.11 along with the control effort that is obtained from

$$u(s) = -\widetilde{p}(s)qq_d(s,\varepsilon)d(s) \tag{4.16}$$

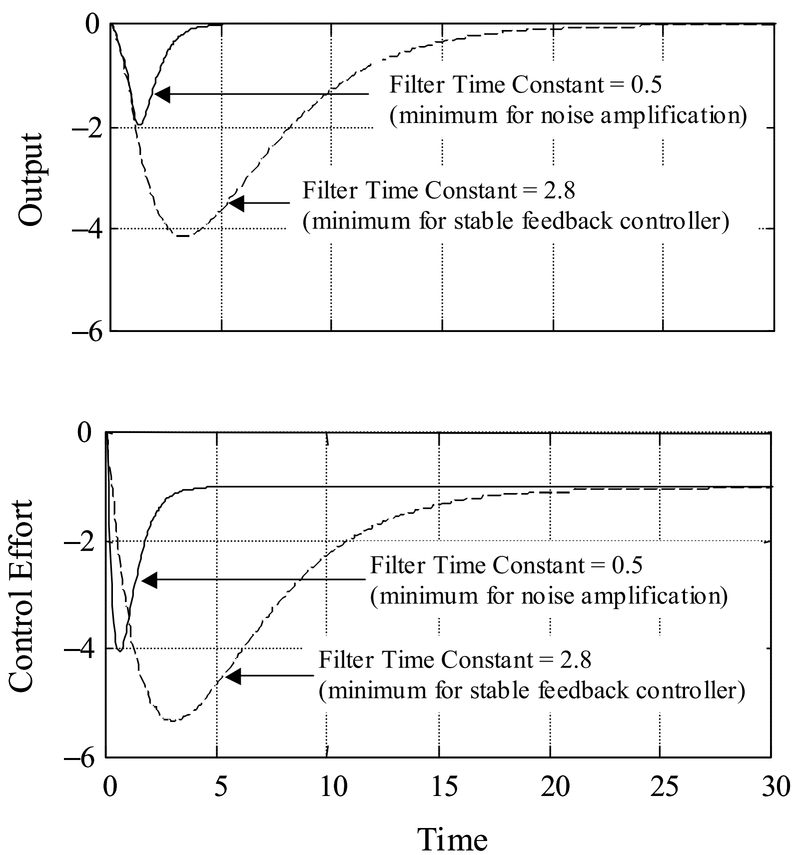

Figure 4.11 Comparison of responses of the unstable process of Example 4.5 for different IMC filter-time constants.

We must emphasize that the response given by Eq. (4.15), which has a filter time constant of 0.5, is the response is that obtained from an internally stable implementation of a 2DF IMC system such as that suggested by Berber and Brosilow (1999). If we attempt to implement the control system via Figure 4.10, then we find that the feedback controller, $c(s)$, associated with a filter-time constant of 0.5 is unstable, as shown in Eq. (4.17). The resulting closed-loop control system also turns out to be unstable.

$$\begin{aligned} c(s) &= qq_d(s,.5)/(1-\tilde{p}(s)qq_d(s,.5)) \\ &= \frac{-.321(.0138s^2+.128s+1)(.0175s^2+.235s+1)(.137s+1)(.512s+1)}{(.00497s^2-.00993s+1)(.0318s^2-.0684s+1)(.0168s+1)s}. \end{aligned} \tag{4.17}$$

To get a stable controller, $c(s)$, it is necessary to increase the filter-time constant from 0.5 to 2.8. The controller is given by Eq. (4.18).

$$\begin{aligned} c(s) &= qq_d(s,2.8)/(1-\tilde{p}(s)qq_d(s,2.8)) \\ &= \frac{-.0316(.0138s^2+.128s+1)(.0175s^2+.235s+1)(.137s+1)(38.25s+1)}{(.00618s^2+.0283s+1)(.0424s^2+4.66\times10^{-5}s+1)(.0312s+1)s}. \end{aligned} \tag{4.18}$$

The closed-loop control system using the controller given by Eq. (4.18) is stable, and the disturbance response is that shown in Figure 4.11. Clearly, there is a significant performance incentive for implementing an internally stable 2DF IMC system for this example.

Note that the responses in Figure 4.11 are different from the responses for stable processes. For a stable process, decreasing the IMC filter-time constant makes the control effort more aggressive. That is, both the magnitude and the speed of response of the control effort are increased. For an unstable process, decreasing the filter-time constant increases the speed of response of the control effort, but the magnitude of the control effort decreases. This difference, as well as other important differences between control systems for inherently stable and inherently unstable processes, changes the method of tuning the control system to account for process uncertainty. Therefore, in Chapter 8, we separate the discussion of tuning control systems for uncertain inherently stable and inherently unstable processes into different sections.

4.5 SOFTWARE TOOLS FOR 2DF IMC DESIGNS

2DF IMC controllers for both stable and unstable systems with perfect models can be designed with the aid of the IMCTUNE software included with this text. The software computes the q_d portion of the feedback controller given the form of q and a filter-time constant. It will also compute the filter-time constant that achieves any noise amplification factor specified in the Default Values window, as described in Appendix F. The software provides step setpoint and disturbance responses for inherently stable and unstable processes using the control system configurations of figures 4.3 and 4.10 respectively. For example, the responses in figures 4.4 through 4.7 and the response for a filter-time constant of 2.8 in Figure 4.10 either were, or could have been, obtained from IMCTUNE.

4.6 SUMMARY

2DF control systems should be used for inherently unstable processes, or for stable processes where the disturbances pass through the process or through a lag whose time constants are on the order of, or larger than, the process lag time constants. There is usually no advantage of a 2DF controller over a 1DF controller when the disturbance lag time constant is small relative to the process lag time constants or when the disturbance passes through a stable process whose lead time constants are larger than its lag time constants (i.e., one or more process zeros lie to the right of the process poles). When in doubt, it is advisable to design a 2DF control system and compare its behavior to that of a 1DF control system. Only when there is a substantial improvement in the disturbance response should the 2DF controller be implemented.

The setpoint filter and $q(s)$ portion of the feedback controller in Figure 4.3 are designed using the 1DF methods in Chapter 3. The $q_d(s,\varepsilon)$ portion of the feedback controller is chosen as

$$q_d(s,\varepsilon,\alpha) = \frac{\sum_{i=0}^{n} \alpha_i s^i}{(\varepsilon s+1)^n}, \tag{4.4}$$

where n is number of poles in $\tilde{p}_d(s)$ to be cancelled by the zeros of $(1-\tilde{p}(s)qq_d(s))$. The constants α_i are chosen to cancel selected poles or other portions of the denominator of the disturbance transfer function $p_d(s)$.

The IMC configurations of figures 4.1 and 4.3 are internally unstable for inherently unstable processes, and therefore cannot be used to control such processes. They can, however, be used for the purpose of analysis and controller design. Either the configuration given in Figure 4.10, or the algorithm suggested by Berber and Brosilow (1999) can be used to control unstable processes.

Problems

4.1 Compare the 1DF and 2DF disturbance responses for problems 3.4 and 3.5 of Chapter 3, assuming that the disturbances pass through the process. Suggestion: For processes whose denominators are higher than second order, design the feedback controller to cancel either the one or two largest time constant poles or the linear or quadratic terms in the denominator.

4.2 Design and implement controllers for each of the following processes. Compare the ideal response using a filter-time constant that yields a noise amplification factor of 20, with the minimum filter-time constant for a stable control system using the configuration of Figure 4.10.

a) $$p_d(s) = p(s) = e^{-s} \big/ (s+1)(s^2 + .1s + 1)$$

b) $$p_d(s) = p(s) = \frac{e^{-s}}{s(s+1)}$$

c) $$p_d(s) = p(s) = \frac{2}{2s-1}e^{-0.5s}$$

d) $$p_d(s) = p(s) = \frac{2(-.5s+1)}{(2s-1)(s+1)}$$

References

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